From QCD to a dynamical quark model: construction and some meson spectroscopy

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We introduce an effective quark model that is in principle dynamically derivable from the QCD action. An important feature is the incorporation of spontaneous chiral symmetry breaking in a renormalizable fashion. The quark propagator in the condensed vacuum exhibits complex conjugate poles, indicative of an unphysical spectral form, i.e. confined quarks. Moreover, the ensuing mass function can be fitted well to existing lattice data. To validate the physical nature of the new model, we identify not only a massless pseudoscalar (i.e. a pion) in the chiral limit, but we also present reasonable estimates for the ρ meson mass and decay constant, employing a contact point interaction and a large N argument to simplify the diagrammatic spectral analysis. We stress that we do not use any experimental input to obtain our numbers, but only rely on our model and lattice quark data.

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Next to confinement — the absence of color charge particles as in the observable QCD spectrum, the other crucial nonperturbative ingredient of QCD is the dynamical breaking of the (almost) chiral symmetry (DχSB). The latter breaking is what ensures most of the mass of the mesons/baryons and what explains the mass gap between the light pseudoscalar mesons and the rest of the spectrum due to the Goldstone boson nature of the former. Albeit confinement and DχSB are well appreciated, their treatment, as deeply nonperturbative phenomena, inspires up to today active research. Here, we present a novel quark model, directly grounded in QCD. We have a few prerequisites to be met, i.e. the model should be (i) able to capture the correct quark dynamics, in particular to describe properly the nonperturbative lattice quark propagator; (ii) renormalizable, as we require to describe quark dynamics over the whole momentum range, including its (well-known) UV behaviour; (iii) practical to compute with; (iv) displaying the correct chiral behaviour such as massless pions in the chiral limit; (v) allowing to construct the rest of the meson spectrum.

We initiate from the QCD action in $4d$ Euclidean space [22], but we add an extra piece to it:

$$S=\int \mathrm{d}^4x\left(\frac{1}{4}F_{\mu\nu}^2+\overline{\psi}D\!\!\!/\psi\underline{-\overline{\lambda}\partial^2\xi-\overline{\xi}\partial^2\lambda-\overline{\eta}\partial^2\theta+\overline{\theta}\partial^2\eta}\right)$$

The additional (fermion) fields are perturbatively trivial, as the underlined (quadratic) piece constitutes a unity. Moreover, it is BRST exact [1]. The previous action is in fact equivalent to the QCD one, in particular should its symmetry content be. We treat the new fermion fields as chiral singlets, as their quadratic form is not of the usual kind [23]. We nevertheless recover the QCD chiral symmetry with the transformations

$$\delta_5\psi=i\gamma_5\psi\,,\quad \delta_5\overline{\psi}=i\overline{\psi}\gamma_5\,,\quad \delta_5(\text{rest})=0. \tag{1}$$

We did not explicitly write the gauge fixing part. As our action is supposed to properly describe nonperturbative gluon and quark dynamics, a suitable nonperturbative gauge fixing as the Gribov-Zwanziger scheme in the Landau gauge can be selected [2]. It is now useful to record the following mass dimensions $\dim[\psi,\overline{\psi}]=3/2$ and $\dim[\lambda,\xi,\eta,\theta,\overline{\lambda},\overline{\xi},\overline{\eta},\overline{\theta}]=1$. Consider next the local composite operators (LCO) $O_1=\overline{\xi}\psi+\overline{\psi}\xi-\overline{\lambda}\psi-\overline{\psi}\lambda$ and $O_2=\overline{\lambda}\xi+\overline{\xi}\lambda+\overline{\eta}\theta-\overline{\theta}\eta$, with $\dim[O_1]=5/2$ and $\dim[O_2]=2$. The mixed fermion condensate $\langle O_1\rangle$ serves as an order parameter for chiral symmetry breaking, indeed $O_1=-\delta_5\pi$ with $\pi=-i\left(\overline{\xi}\gamma_5\psi+\overline{\psi}\gamma_5\xi-\overline{\lambda}\gamma_5\psi-\overline{\psi}\gamma_5\lambda\right)$. We shall later on show that this π corresponds to the pion field, viz. a massless pseudoscalar in the here considered chiral limit. Let us now construct the underlying effective action so that the nonperturbative dynamics related to $\langle O_1\rangle$ can be taken into due account. We introduce 2 scalar sources, J and j, coupled to O_1 and O_2, with $\dim[J]=3/2$, $\dim[j]=2$,

$$S\to S+\int \mathrm{d}^4x\left(JO_1+jO_2-\zeta(g^2)\frac{j^2}{2}\right) \tag{2}$$

as acting with a derivative w.r.t the sources allow to define the operators at the quantum level. The parameter ζ must be introduced to allow for a homogeneous linear renormalization group for the effective potential, while its value can be consistently determined order by order, making it a function of the coupling g^2. It reflects the vacuum energy divergence $\propto j^2$. We refer to [3] for the seminal paper plus toolbox. For the current work, we shall not need the explicit effective potential. As an important asset of $4d$ quantum field theory is its multiplicative renormalizability —to have a clean-cut way to deal with UV infinities—, this property needs to be established for eq. (2). Using a more general set of sources, leading to an extensive list of Ward identities to be obeyed by the $1PI$ generating functional including the LCOs, this can be proven to all orders of perturbation theory [1], from where [24] it also becomes evident that there is no pure vacuum term in J. Having assured ourselves of the controllable UV nature of our effective action, we still need a workable version of it. We employ a classical trick: we introduce 2 scalar fields via a double Hubbard-Stratonovich (HS) unity into the partition function,

$$1=\mathcal{N}\int[\mathrm{d}\sigma d\Sigma]\,e^{-\int \mathrm{d}^4x(\sigma-\zeta j+O_2)^2}e^{-\frac{1}{2\Lambda}\int \mathrm{d}^4x(\Sigma-\Lambda J+O_1)^2}\,, \tag{3}$$

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which leads to the following equivalent action, instead of (2),

$$
\begin{aligned}
S_f + S_{j,J} \;&\equiv\; \int \mathrm{d}^4x \left(\frac{1}{4}F_{\mu\nu}^2 + \overline{\psi}D\!\!\!/\psi - \overline{\lambda}\partial^2\xi - \overline{\xi}\partial^2\lambda \right.\\
&\left. -\overline{\eta}\partial^2\theta + \overline{\theta}\partial^2\eta + \frac{\sigma^2}{2\zeta} + \frac{\sigma}{\zeta}O_2 + \frac{O_2^2}{2\zeta} + \frac{\Sigma^2}{2\Lambda} + \frac{O_1^2}{2\Lambda} + \frac{\Sigma}{\Lambda}O_1 \right)\\
&+ \int \mathrm{d}^4x \left(-\sigma j - \Sigma J + \Lambda J^2 \right) . \qquad (4)
\end{aligned}
$$

Here Λ is a mass dimension 1 parameter, the introduction of which is necessary to end up with the appropriate mass dimensions throughout, given that $\dim[\sigma] = 2$, $\dim[\Sigma] = 5/2$. Notice that Λ will not enter any physical result at the end if we were to compute exactly, as indeed the underlying transformation (3) constitutes nothing more than a unity. Acting with $\frac{\partial}{\partial\{j,J\}}\Big|_{j=J=0}$ on both versions of the partition function, i.e. before and after the HS transformation, simply provides with the correspondences $\langle\Sigma\rangle = -\langle O_1\rangle$ and $\langle\sigma\rangle = -\langle O_2\rangle$.

Let us now collect a few important observations. (i) In a loop expansion approach, the parameter Λ will unavoidably enter any calculated quantity. However, under the assumption that passing to higher and higher order one gets closer and closer to the exact result, which encompasses Λ-independence, we can fix Λ in a case-by-case scenario by relying on the principle of minimal sensitivity (PMS) [4]: we must look for solutions of $\frac{\partial E_{vac}}{\partial\Lambda} = 0$ or higher derivatives if the latter eq. has no zeros. (ii) The effective potential itself could be examined using the background field method [5]; then we do no longer need the sources and can set them to zero, i.e. we can drop the $S_{j,J}$ part in the action (4). (iii) If the dynamics of the QCD action would decide that $\langle\Sigma\rangle = \langle\sigma\rangle = 0$, then we are again dealing with nothing else than QCD without DχSB, as the trivial and thus irrelevant unities can be integrated out. If, on the contrary, $\langle\Sigma\rangle \neq 0$ by means of dimensional transmutation, we find ourselves in a vacuum where chiral invariance is dynamically broken. We draw attention to the crucial invariance of the action itself, $\delta_5 S_f = 0$, since we naturally have $\delta_5\Sigma = -\delta_5 O_1$. The rôle of $\langle\sigma\rangle$ is to furnish the dynamical mass for the auxiliary fermion fields, the "m^2" of [1]. (iv) One might wonder about the connection with the standard chiral condensate, $\langle\overline{\psi}\psi\rangle$? This can be straightforwardly inferred to be $\langle\overline{\psi}\psi\rangle \propto \langle\Sigma\rangle$, for example by adding a mass term μ to $\overline{\psi}\psi$ in (4) and deriving the vacuum energy w.r.t. μ at the end whilst setting $\mu = 0$. This illustrates clearly the 1-1 correspondence between $\langle\Sigma\rangle$, $\langle\overline{\psi}\psi\rangle$ and chiral symmetry breaking. (v) With a bare quark mass μ, the tree level quark propagator yields

$$
\langle\overline{\psi}\psi\rangle_p = \frac{ip\!\!\!/ + \mathcal{M}(p^2)}{p^2 + \mathcal{M}^2(p^2)} , \quad \mathcal{M}(p^2) = \frac{\langle\Sigma\rangle/\Lambda}{p^2 + \langle\sigma\rangle} + \mu \qquad (5)
$$

One should observe the momentum dependent mass function $\mathcal{M}(p^2)$ as a result of the chiral symmetry breaking. Precisely this tree level functional form has been applied in fits to lattice Landau gauge quark propagators in [7, 8]. These observations corroborate the relevance of our model, since we end up with a quark propagator in a nontrivial vacuum, which functional form is consistent with the nonperturbative lattice counterpart. Moreover, we did not make any sacrifices w.r.t. the renormalizability and in the absence of condensation, our action is exactly equivalent to QCD. For further usage, we will take advantage of these lattice studies and employ the fitting parameters to fix a value of our condensates. This will provide us with a tree level quark propagator, with the global form factor $\mathcal{Z} \equiv 1$. Including loop corrections on top of the nonperturbative vacuum will lead to a nontrivial $\mathcal{Z}(p^2)$, as also seen in e.g. FIG. 5 of [8], which however only deviates mildly from 1 over a large range of momenta. The tree approximation $\mathcal{Z} = 1$ thus appears to be a valid one. We shall hence not attempt here to self-consistently derive and minimize the effective action to compute $\langle\Sigma\rangle$ and $\langle\sigma\rangle$, which is a technically challenging task beyond the scope of this work [3]. (vi) As a final important remark, let us scrutinize if we can introduce a pion field. We take $\langle\Sigma\rangle \neq 0$ and consider the already introduced field π [we assume the chiral limit here, $\mu = 0$]. Using S_f, the chiral current is easily derived to be $j_\mu^5 = \overline{\psi}\gamma_\mu\gamma^5\psi$. We can then follow more or less the standard derivation, presented in e.g. [6]: the correlation function $\mathcal{G}_\mu(x-y) = \langle j_\mu^5(x)\pi(y)\rangle$ is subject to $\partial_\mu\mathcal{G}_\mu(x-y) = \delta(x-y)\langle\Sigma\rangle$, as can be shown by either using the path integral or a current algebra argument as in [6]. Fourier transforming and using the Euclidean rotational invariance learns that $\mathcal{G}_\mu = \frac{p_\mu}{p^2}\langle\Sigma\rangle$. To close the argument, we consider the $\mathcal{S}$-matrix element of the current destroying a pion state, $\langle j_5^\mu(x)\pi(p)\rangle \propto p_\mu e^{-ipx}$ which is related to the (pion amputated) propagator when the pion is put on-shell. Assuming the pion has a mass m_π^2, we would get $\langle j_5^\mu(x)\pi(p)\rangle \propto \lim_{p^2\to -m_\pi^2}(p^2+m_\pi^2)\mathcal{G}_\mu(p)e^{-ipx}$ with pion propagator $\langle\pi\pi\rangle_{p^2\sim -m_\pi^2} \propto \frac{1}{p^2+m_\pi^2}$. Recombination immediately provides us with $\lim_{p^2\to -m_\pi^2}(p^2+m_\pi^2)\mathcal{G}_\mu(p) \propto p_\mu$, and since we have already shown that $\mathcal{G}_\mu = \frac{p_\mu}{p^2}\langle\Sigma\rangle$, we must require $m_\pi^2 = 0$, i.e. the field π does describe a massless particle. It is instructive to notice that, using the tree level action stemming from (4) in the condensed phase, we can rewrite $\pi \propto \overline{\psi}\frac{\Sigma/\Lambda}{-\partial^2+\langle\sigma\rangle}\gamma^5\psi$ by means of the equations of motions of the auxiliary fermions. We recognize here a nonlocal version of the usual pseudoscalar pion field. This is not a surprise, since the action (4) itself becomes a nonlocal quark action upon integrating out the extra fermion fields in the condensed phase.

As the proposed framework displays the desired chiral properties, we may address further its meson spectrum. As a representative example, let us construct and solve a gap equation for the (charged) $\rho^\pm$ meson mass under suitable simplifying approximations. Using the data of [8], in case their FIG. 5, we consider degenerate up (u) and down (d) quarks with current mass $\mu = 0.014$ GeV. The dynamical quark mass can be fitted excellently with $M(p^2) = \frac{M^3}{p^2+m^2} + \mu$ with $M^3 = 0.1960(84)$ GeV^3, $m^2 = 0.639(46)$ GeV^2 with χ^2/d.o.f. = 1.18, see FIG. 1. We need $\rho_\mu^- = \overline{u}\gamma_\mu d$, $\rho_\mu^+ = \overline{d}\gamma_\mu u$ with correlator

$$
\langle\rho_\mu^-\rho_\nu^+\rangle_k = \frac{1}{3}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\langle\rho_\alpha^-\rho_\alpha^+\rangle_k , \qquad (6)
$$

keeping in mind that in the degenerate case, it is transverse

thanks to the EOMs and therefore guaranteed to describe the 3 polarizations of a massive spin 1 particle. We are ultimately interested in obtaining a pole in the above channel, corresponding to a bound state. We rely on the technology set out in [9], adapted to the QCD case. Specifically, we shall first be concerned with the one-loop contribution to the correlation function,

$$\langle\rho^-_\mu\rho^+_\nu\rangle_k = 8\int\frac{\mathrm{d}^4q}{(2\pi)^4}f^{-+}(k,q)/\Big[(k-q)^2\left((k-q)^2+m^2\right)^2 + \left(M^3+\mu\left((k-q)^2+m^2\right)\right)^2\Big]\Big[q^2\left(q^2+m^2\right)^2+\big(M^3+ \mu(q^2+m^2)\big)^2\Big]\,,\quad f^{-+}(k,q)=(q\cdot(k-q))\left((k-q)^2+m^2\right)^2 \times\left(q^2+m^2\right)^2+2\left(M^3+\mu\left((k-q)^2+m^2\right)\right)\left((k-q)^2+m^2\right) \times\left(M^3+\mu\left(q^2+m^2\right)\right)\left(q^2+m^2\right) \tag{7}$$

We can decompose the appearing denominator into $R/(q^2+\omega)+R_+/(q^2+\omega_r+i\theta)+R_-/(q^2+\omega_r-i\theta)$ by using the poles $y_0=-\omega$, $y_\pm=-\omega_r\pm i\theta\equiv-\omega_\pm$ of the cubic equation $y(y+m^2)^2+\left(M^3+\mu(y+m^2)\right)^2=0$. In appropriate GeV units: $R\approx 2.467$, $\omega\approx 0.849$, $R_\pm\approx -1.234\pm i\,15.121$, $\omega_\pm = 0.214\pm i\,0.052$. With the previous numbers filled in, we encounter a real pole and a pair of *cc* poles, which correspond to the so-called *i*-particles of the seminal works [10] where it has been discussed how a pair of such poles can be combined to give a physical, i.e. consistent with the Källén-Lehmann (KL) representation, contribution to the spectral form of a bound state propagator [25]. For the remainder of this work, we shall thence only be concerned by this physical part of the correlator, under the assumption that any unphysical piece, originating from combining poles in pairs that are not *cc*, will eventually cancel out when a full-fledged analysis and methodology to deal with the *i*-particles would become feasible. Explicitly, we can extract the spectral representation using the complex mass cut-rule technology of [11]. A rather tedious computation leads to

$$\langle\rho^-_\alpha\rho^+_\alpha\rangle_k=\int_0^\infty \mathrm{d}\tau\frac{\rho^{-+}(\tau)}{\tau+k^2}+\text{unphysical}\,,\quad \rho^{-+}(\tau)\geq 0$$
$$\rho^{-+}(\tau)=\theta(\tau-\tau_1)\frac{R^2}{\pi^2}\sqrt{1/4-\omega/\tau}\Big[(\tau/2-\omega)\left(\omega+m^2\right)^4 +2\left(M^3+\mu(\omega+M^2)\right)^2\left(\omega+m^2\right)^2\Big]+\theta(\tau-\tau_2)\frac{R_+R_-}{\pi^2} \times\frac{\sqrt{(\tau-2\omega_r)^2-4(\omega_r^2+\theta^2)}}{\tau}\Big[(\tau/2-\omega_r)\left((\omega_r+m^2)^2+\theta^2\right)^2 +2\Big(\left(M^3+\mu(\omega_r+m^2)\right)^2+\mu^2\theta^2\Big)\left((\omega_r+m^2)^2+\theta^2\right)\Big] \tag{8}$$

with thresholds $\tau_1=4\omega$, $\tau_2=2\omega_r+2\sqrt{\omega_r^2+\theta^2}$. Eq. (8) is the spectral representation of the single bubble approximation to the ρ-correlator. In order to find a bound state, we need to take into account the QCD interaction structure. The quarks do not interact directly, the force being mediated by the gluon. Unfortunately, taking the full gluon interaction into account appears to be a hopelessly complicated task. To simplify the analysis to get a tractable problem in this first attempt, we consider the observation made in e.g. [12] that a gluon contact point interaction can give qualitatively good results. Specifically, it is by now well accepted that the Landau gauge gluon propagator $\mathcal{D}(p^2)$ becomes dynamically massive-like in the infrared. We make the assumption that the gluon is massive-like in the region $p<$ 1-2 GeV, which is anyhow the region where the relevant QCD physics is supposed to happen and that it can be approximated by a constant in momentum space, $\mathcal{D}(p^2)=\frac{1}{\Delta^2}$, or by $\mathcal{D}(x-y)\propto\delta(x-y)$ in position space. Integrating out at lowest order such gluon leads, via the Dirac delta behaviour, to a NJL-like (contact) interaction between the quarks, more precisely one finds after some Fierz rearranging, see the Appendix of [13], the effective interaction $\frac{1}{2}G(\overline{q}\blacklozenge q)^2+$ lower in $1/N$, where $\blacklozenge\in\{1,i\gamma^5,\frac{i}{\sqrt{2}}\gamma_\mu,\frac{i}{\sqrt{2}}\gamma_\mu\gamma_5\}$ and $G=\frac{2g^2}{\Delta^2}\frac{N^2-1}{N}$. To allow for additional simplification without throwing away too much crucial dynamics, we may furthermore consider only the leading order in $1/N$. For the eventual effective coupling G, we could borrow the various NJL estimates from [14], where $G\sim$ 6-10 GeV^{-2}. We can however also estimate more directly an appropriate value. In the Landau gauge, the gluon and ghost propagator form factors, $\mathcal{Z}_{gl}(p^2,\overline{\mu}^2)$ and $\mathcal{Z}_{gh}(p^2,\overline{\mu}^2)$, can be used to define a renormalization scale $\overline{\mu}$ independent strong coupling constant, $4\pi g^2(p^2)\equiv\alpha(p^2)=\alpha(\overline{\mu}^2)\mathcal{Z}_{gl}(p^2,\overline{\mu}^2)\mathcal{Z}^2_{gh}(p^2,\overline{\mu}^2)$, see e.g. [15]. Using the most recent available lattice data on this matter [16], albeit for $N=2$ without fermions, which rely on a MOM scheme ($\mathcal{Z}_{gh}=\mathcal{Z}_{gl}=1$ at $p^2=\overline{\mu}^2$, with $\overline{\mu}=2.2$ GeV), we can estimate from their FIG. 5 that $\alpha(\overline{\mu})\sim 0.5$. For the "constant" MOM scheme gluon propagator, we may simply set $\Delta^2=\overline{\mu}^2$, thereby overestimating the UV and underestimating the IR, to roughly approximate $G\sim 8.5$ GeV^{-2}, quite nicely in the NJL ballpark. For this paper, we will use $G=5, 7.5, 10$ GeV^{-2} as exemplary values. In the large-N approximation, we can then consistently consider the sum of bubble diagrams [26] in our quark model, see FIG. 2.

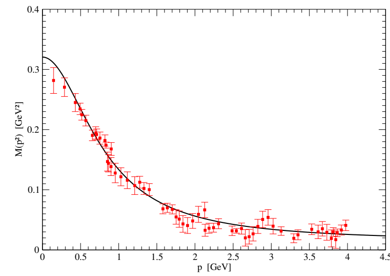

FIG. 1: Lattice quark mass function [8] with its fit $\mathcal{M}(p^2)$.

FIG. 2: Bubble diagrams for the meson correlator. Full (open) circles represent four-fermion vertices (meson operator insertions); solid lines are nonperturbative quark propagators.

The four-fermion coupling includes *a priori* all interaction channels $\blacklozenge\in\{1,i\gamma^5,\frac{i}{\sqrt{2}}\gamma_\mu,\frac{i}{\sqrt{2}}\gamma_\mu\gamma_5\}$, while the initial and final

insertions carry the vectorial character of the ρ meson. In this case, as well-known, only the vector four-fermion coupling γ_μ contributes [27] and the resummation of the bubble chain reduces to a geometric series involving the one-loop result, (8). Using the already derived spectral form, we then get

$$\mathcal{R}^{-+}(k^2) = \frac{\mathcal{F}^{-+}(k^2)}{1+\frac{G}{2}\mathcal{F}^{-+}(k^2)}, \quad \mathcal{F}^{-+}(k^2) = \int_0^\infty \frac{\rho^{-+}(\tau)\mathrm{d}\tau}{\tau+k^2}.$$

An important observation is that the branch cut structure of $\mathcal{R}^{-+}(k^2)$ is completely determined by that of $\mathcal{F}^{-+}(k^2)$, in particular $\mathcal{R}^{-+}(k^2)$ has a physical KL form if $\mathcal{F}^{-+}(k^2)$ does. The remaining task at hand is to identify if $\mathcal{R}^{-+}(k^2)$ allows for poles in the physical region $\max(-\tau_1,-\tau_2) < k^2 < 0$. Therefore, we need to solve the gap equation $\mathcal{F}^{-+}(k^2) = -2/G$. As common for $4d$ quantum field theories, the quantity $\mathcal{F}^{-+}(k^2)$ is divergent due to violent UV behaviour, explicitly visible from the integral representation (8). We assume the use of dimensional regularization and Landau gauge renormalization factors to kill off sub-loop divergences, whereas the residual infinities in the composite operator Green function can be taken care off by additive subtractions in the BPHZ approach. At the level of the KL integral (8), $n > 0$ subtractions at scale $\mathcal{T} > \max(-\tau_1,-\tau_2)$ corresponds to

$$\mathcal{F}_{sub}^{-+}(k^2,\mathcal{T}) = (\mathcal{T}-k^2)^n \int_0^\infty \mathrm{d}\tau \frac{\rho^{-+}(\tau)}{(\tau+\mathcal{T})^n(\tau+k^2)}. \qquad (9)$$

In the current case at least 2 subtractions are required. Notice that if no subtractions were to be necessary, then $\mathcal{F}^{-+}(k^2)$ is a strictly decreasing function thanks to $\rho^{-+}(\tau) \geq 0$, so at most one solution is possible. This property is not necessarily maintained at the subtracted level, except for $n = 1$. Consequently, sometimes spurious extra solutions appear, caused by the enforced functional behaviour after subtraction. For this reason, we take $n = 3$ in which case only a single solution is found. Next to the pole of the propagator, also the associated residue carries physical information. With the conventions of [18, 19], we define the decay constant $f_{\rho^\pm}$ via $f_{\rho^\pm} m_{\rho^\pm} \varepsilon_\mu = \langle 0|\overline{u}\gamma_\mu d|\rho^+\rangle$, with ε_μ the polarization tensor of the ρ^+ meson, normalized as $\varepsilon_\mu \cdot \varepsilon_\mu = 3$. Using the matrix element representation of the KL spectral density, it becomes clear that $3f_{\rho^\pm}^2 m_\rho^2$ is nothing else than the residue of $\mathcal{R}^{-+}(k^2)$ at its pole $-m_{\rho^\pm}^2$. In FIG. 3, we have displayed both $m_\rho^\pm(\mathcal{T})$ and $f_{\rho^\pm}(\mathcal{T})$. To get a grip on a reasonable value for $\mathcal{T}$, we rely again on PMS as observable quantities should not depend on a chosen subtraction scale. Since we have two (related) physical quantities at hand, we minimized $\delta(\mathcal{T}) = |\overline{m}'_{\rho^\pm}(\mathcal{T})| + |\overline{f}'_{\rho^\pm}(\mathcal{T})|$ where the prime means derivation w.r.t $\mathcal{T}$, and we used the rescaled mass and decay constant, $\overline{m}_{\rho^\pm}(\mathcal{T}) = m_{\rho^\pm}(\mathcal{T})/m_{\rho^\pm}(-\tau_2)$, $\overline{f}_{\rho^\pm}(\mathcal{T}) = f_{\rho^\pm}(\mathcal{T})/f_{\rho^\pm}(-\tau_2)$ to attribute to both quantities an "equal start". We choose the smallest possible scale, viz. $\mathcal{T} = -\tau_2$, as reference. FIG. 3 shows that $\delta(\mathcal{T})$ develops a clear minimum for all values of the effective coupling. Specifically, we find $\mathcal{T}_*^{G=5,\ 7.5,\ 10} \approx -0.38, -0.43, -0.46$ GeV leading to $m_{\rho^\pm}^{G=5,\ 7.5,\ 10} \approx 0.84, 0.83, 0.83$ GeV and $f_{\rho^\pm}^{G=5,\ 7.5,\ 10} \approx 0.13, 0.10, 0.09$ GeV. We notice that the mass estimate is pretty stable, while the decay constant seems to be more sensitive to the value of the coupling. We found it useless to attempt presenting a detailed error analysis given the various deliberately rude approximations, but we find it at least reassuring that we do find a result not too far off the experimental ρ meson mass, 775.49 ± 0.34 MeV [17], despite the approximations made, next to having used heavier bare quarks. For the decay constant, we can quote experimental and lattice estimates, $f_{\rho^\pm}^{exp} \approx 0.208$ GeV [20], $f_{\rho^\pm}^{latt} \approx 0.25$ GeV [18]. Our estimates, rude as they may be, are somewhat lower, though it must be remarked that the heavier the particle mass, the smaller the decay will get for a fixed value of the residue. If we, for example, fix $\mathcal{T}$ to its value at $G = 5$ GeV^2 where $m_{\rho^\pm}$ equals its experimental value, we would obtain $f_{\rho^\pm} \approx 0.16$ GeV, whose deviation from the experimental value is of the same size as its lattice counterpart.

Summarizing, we presented a calculable quark model with the right properties to describe the mesonic QCD sector. In future work, it would, for example, be interesting to (i) compute the effective potential to get self-consistent estimates for the condensates, (ii) consider other meson states and their properties, (iii) derive the Gell-Mann-Oakes-Renner like relations for multiflavour versions with bare quark masses, etc. Given that our model incorporates an effective way of confining quarks simultaneously with breaking the chiral symmetry, thereby generating propagators that are consistent with lattice QCD, it might provide an interesting setup to investigate nontrivial finite temperature dynamics [21], given the intertwining of (de)confinement and chiral symmetry breaking/restoration.

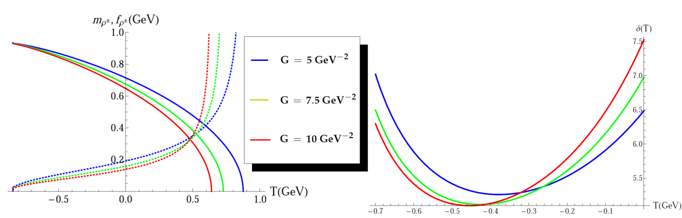

FIG. 3: $m_{\rho^\pm}$ (full line) and $f_{\rho^\pm}$ (dashed line) in terms of $\mathcal{T}$ (left) and $\delta(\mathcal{T})$ (right) (see main text for definitions).

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[22] We did not write bare quark masses. For the simplicity of presentation, we restricted ourselves to a single flavour.
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[24] The operator O_1 is split in 4 pieces which are separately studied using 4 sources, see eqns. (25)-(27) in [1]. From eq. (72), it can be immediately inferred that these 4 operators share their renormalization factor, meaning that the a priori 4 independent bare sources can be taken equal to our J_1.
[25] In the last Ref. of [10], a reasonable light glueball mass spectrum was constructed using an new infrared moments technique.
[26] In conventional Bethe-Salpether studies of meson propagation, such resummation is also known as the Random Phase Approximation (see e.g. [13]).
[27] Indeed, it is straightforward to show that bubbles that connect a vector insertion γ^μ with a coupling in a different channel (i.e. $1, \gamma^5, \gamma_\mu\gamma_5$) are absent: they either vanish identically due to Dirac algebra and integration symmetries or produce a term $\propto k^\mu f(k^2)$, which does not contribute to the transverse correlator (cf. (6)) under consideration.